MATH 5530. Post-modern algebra

This is the second semester of the graduate algebra sequence. We will use a post-modern epistemologico-genetic approach, in which concepts are introduced by tracing their historical development. The two topics are

Modules over non-commutative rings. This is done by studying the representation theory of finite groups over a field of characteristic zero as done by G. Frobenius and I. Schur. When we think of representation theory as a decomposition theory for modules over the group algebra, we recover, in their original form, the concepts of module theory.

Finite field and Galois theory. Here we take the rediscovery of finite fields by electrical engineers developing linear error-correcting codes as the historical context. We study the basic theory of finite fields and how to construct them using irreducible polynomials and in particular, we regard a finite field as the splitting field of the polynomial

$$x^{p^n} - x.$$

We will use this theory to construct the Hamming codes and the $(24, 12)$-Golay code. Error-correcting codes make ALL modern digital communicationpossible and is a good example of the unacknowledged power and glory of mathematics.

There are no specific texts for this course. Students are encourage to discover the sources on their own and indeed, to reinvent the subject. For this reason, homeworks are individualized; students are expected to find their own topics depending on their personal interest and study them, reporting on their research in reports. Evaluation and grading are an interactive negotiation between the student and instructor.